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Modeling the Monthly Average of the Quota Values per AFP and Type 2 Fund with the Box and Jenkins or ARIMA Methodology

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ABSTRACT

The economic and financial crises in the world are recurrent due to the presentation of different patterns. These crises have affected the returns of the private pension system in Peru and there were no effective responses from the Pension Fund Administrators (AFPs). By using the Box and Jenkins or Autoregressive Integrated Moving Average (ARIMA) methodology, the behavior of the monthly average returns of the daily quota values of the type 2 fund—which began in December 2005—of each AFP can be described and forecast. Type 2 funds are distributed 55% in fixed income and 45% in equities, with a balanced profile destined for workers between 45 and 60 years old. The data type of the monthly average returns of the type 2 fund corresponds to the weak stationary time series, since the first moments such as the mean and the variance and autocovariance are time-invariant.

Keywords: time series; profitability; weak stationarity; unit root; white noise.

INTRODUCTION

The Box and Jenkins or Autoregressive Integrated Moving Average (ARIMA) methodology was used to describe and forecast the behavior of the returns of the monthly average of the quota values of the type 2 fund—which began in December 2005—of each AFP. The type of data of the monthly returns of the type 2 fund corresponds to time series and, in order to be modeled with ARIMA, they must be weak-sense stationary, where the first two moments as the mean, and the variance and autocovariance must be time-invariant.

In 1976, Box and Jenkins formalized the Box-Jenkins methodology and ARIMA models (also known as Box-Jenkins models) where they mentioned time series that are supported by stochastic processes (Box, Jenkins & Reinsel, 2008). When forecasting with an ARIMA model, the following steps need to be followed:

1. Identification
2. Estimation
3. Diagnostic checking
4. Forecasting

These steps are shown in Box et al. (2008), where they are referred to as “Stages in the iterative approach to model building” (p.18).

Gujarati and Porter (2010) argue that when a time series is not stationary, the mean and variance are not time-invariant. Moreover, when referring to the Box and Jenkins methodology ARIMA, they add the following:

> The publication by Box and Jenkins of *Time Series Analysis: Forecasting and Control* (op. cit.) ushered in a new generation of forecasting tools. Popularly known as the Box–Jenkins (BJ) methodology, but technically known as the ARIMA methodology, the emphasis of these methods

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> is not on constructing single-equation or simultaneous-equation models but on analyzing the probabilistic, or stochastic, properties of economic time series on their own (...). Unlike the regression models, in which Y_t is explained by *k* regressors X_1, X_2, X_3, ..., X_k, the BJ-type time series models allow Y_t to be explained by past, or lagged, values of *Y* itself and stochastic error terms. For this reason, ARIMA models are sometimes called *atheoretic* models because they are not derived from any economic theory. (pp. 774-775).

Court and Rengifo (2011) state that *El concepto de estacionariedad tiene dos versiones: la estacionariedad estricta y la estacionariedad débil* [The concept of stationarity has two versions: strict stationarity and weak stationarity] (p. 400); each of these is shown below:

Strict Stationarity. It is a stochastic process $\{y_i\}$ with i = 1, 2, …, T. It is strictly stationary if, for a finite real number R and for any set of subscripts i_1, i_2, …, i_T, it is defined as follows:

$$F_{y_{i_1}, y_{i_2}, \ldots, y_{i_T}}(y_1, \ldots, y_T) = F_{y_{i_{1+r}}, y_{i_{2+r}}, \ldots, y_{i_{T+r}}}(y_1, \ldots, y_T)$$

Weak Stationarity. It is a stochastic process $\{y_i\}$ with i = 1, 2, …, T. It is weakly stationary if the meets the following:

$$E(y_i) = \mu$$
$$E(y_i - \mu)^2 = \sigma^2 < \infty$$
$$cov(y_i, y_{i-j}) = \gamma_{i-(i-j)} = \gamma_j$$

Ramon and Lopez (2016) also identify two types of stationarity: strong-sense and weak-sense. For the first case, the four moments of the joint distributions are time-invariant, and for the second, only the first 2 moments are. In this case, the Box and Jenkins methodology is based on weak-sense stationarity.

Problematic Situation

Ñaupas et al. (2014) indicate that in daily life there are repetitive patterns, with certain different characteristics, and that the prediction of natural phenomena is more accurate than social phenomena:

> *Así por ejemplo, conociendo las leyes de Kepler, que explican los movimientos de traslación de los planetas, satélites, cometas y asteroides es posible calcular la ocurrencia de eclipses, mareas y acercamiento de cometas a la órbita de la Tierra. La predicción del tiempo, de inundaciones, terremotos, huracanas, erupciones volcánicas, la ocurrencia de mareas, o de pandemias son más confiables que las ocurrencias de revoluciones, conflictos sociales, golpes de estado, etc.* [Thus, for example, knowing Kepler's laws, which explain the translational movements of planets, satellites, comets and asteroids, it is possible to calculate the occurrence of eclipses, tides and approach of comets to the Earth's orbit. The prediction of the weather, floods, earthquakes, hurricanes, volcanic eruptions and the occurrence of tides, or pandemics are more reliable than the occurrence of revolutions, social conflicts, coups d'état, etc.]. (section 2.4.2. *¿Qué es la investigación natural?*)

These phenomena, that originate crises, impact economies and finances in a negative way, which is why Mira (2016) considers the recurrent occurrence of financial crises. These crises have repeatedly affected the returns of the private pension system in Peru.

The International Labor Organization (ILO) established in 1933 the Convention on Old-Age Insurance, and in 1952 determined the guidelines for old-age benefits. In Peru the National Pension System (SNP), which is currently administered by the Pension Standardization Office (ONP) operated in the beginning. Between 1981 and 2014, as noted by Ortiz, Durán-Valverde, Urban, Wodsak, and Yu (2019), about 30 countries fully or partially privatized their mandatory public pensions, a fact that occurred in Peru in 1993. The Asociación de Administradoras de Fondo de Pensiones[2] (2018) defines the pension in the Peruvian private pension system (SPP) as *el ingreso periódico que recibe el afiliado como consecuencia de un proceso previo de suavización de consumo, a través del ahorro a lo largo de su vida laboral en su cuenta de capitalización individual (CIC)* [the periodic income received by the member as a consequence of a previous process of consumption smoothing, through savings throughout his working life in his individual capitalization account (CIC)] (p. 8) with the purpose of ensuring that the retired worker does not face economic difficulties.

AFPs are responsible for managing the contributions of each individual during his working life, that is, they invest their savings in order to obtain a return so that, once retired, the individual can enjoy their contributions and earnings with no need to depend on their family or the State. However, Cruz-Saco et

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al. (2014) pointed out that the pension system in Peru was *ineficiente, tiene una baja probabilidad de incrementar apreciablemente la cobertura en los siguientes 36 años, y presenta, además, un conjunto de inequidades en la asignación de los beneficios previsionales* [inefficient, has a low probability of appreciably increasing coverage in the next 36 years, and also presents a set of inequities in the allocation of pension benefits] (p. 2).

Flórez (2014) also adds that *los ahorros para la jubilación de millones de personas se encuentran expuestos, de manera intrínseca, al comportamiento favorable, así como adverso, de los mercados financieros* [the retirement savings of millions of people are intrinsically exposed to the favorable and adverse behavior of financial markets] (p. 121). These situations are the cause of high volatility, especially when there is more negative news than positive, so an asymmetric behavior of the market, especially the equity market, is observed.

Ortiz et al. (2019) argue that *Los trabajadores se convirtieron así en consumidores obligados del sector financiero, con lo que asumían individualmente todos los riesgos del mercado financiero sin contar con la suficiente información para tomar decisiones sensatas* [Workers became forced consumers of the financial sector, thus individually assuming all the risks of the financial market without enough information to make reliable decisions] (p. 803). In other words, when the market is stable or when there is good news, returns will be positive. On the other hand, according to Yang et al. (as cited in Gutiérrez et al., 2017), financial crises have been characterized by the increase of risk and high volatility, which has negatively affected returns. In the case of the SPP, as a consequence of negative news, the high expectations the SPP initially generated were diluted as the years went by because it did not produce the expected results.

Carlos Palomino, in an interview with RTV San Marcos - UNMSM (2020), stated that the investments of AFPs go into stock-market mechanisms and not into tangible assets. These stock-market instruments are volatile due to economic shocks or cycles which, in turn, are a consequence of external variables, such as, for example, a pandemic.

It should be noted that in November 2006, the absorption of AFP Unión Vida by Prima AFP was authorized. In April 2013, AFP Horizonte was acquired by AFP Integra and Profuturo (50% each). AFP Habitat began operations in April 2013.

The type 2 fund began operations in December 2005. Its investments are distributed 55% in fixed income and 45% in equities, with a balanced profile aimed at workers aged 45 to 60. Figure 1 shows

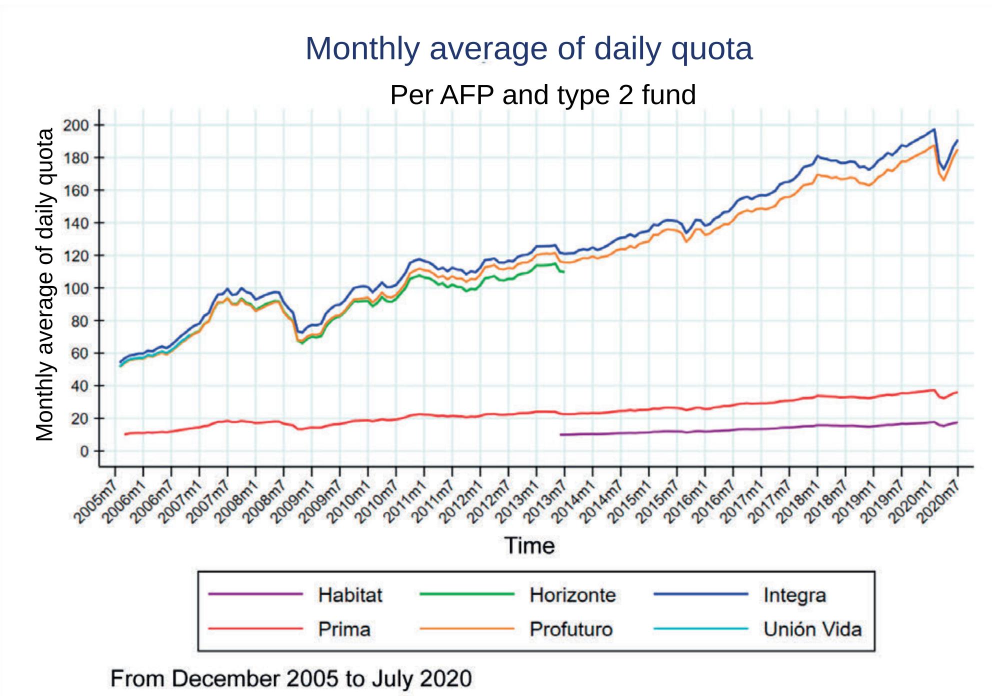

Figure 1. Monthly average of quota values for each AFP and type 2 fund.

Source: Prepared by the author using Stata 16.

a slight upward trend with a sharp drop during the world crisis between 2007 and 2008; a slight drop in April 2011 and 2016 during the first electoral rounds, as well as between 2018 and 2019, which culminated with the dissolution of the Congress of the Republic of Peru; and a sharp decline in March 2020 as a result of COVID-19. This performance corresponds to the monthly average in soles of the daily quota values used for calculating the profitability of the AFPs of type 2 fund.

This work is very useful for workers between 45 and 60 years old, since their investments, managed by the AFPs, are affected by the economic and financial crises in the world. As these crises affect the returns of the Peruvian private pension system, the savings of workers are affected at the time of their retirement. Why did the response to recurrent financial risks adopted by the risk managers of AFPs not mitigate the loss of investment returns of workers? That is the big question asked by workers. Using the Box and Jenkins or ARIMA methodology, the behavior of the monthly average of the daily values of the type 2 fund for each AFP is described, as well as their forecast.

The objective of this research is to determine how to adequately model the monthly average of the quota values for each AFP of the type 2 fund with the Box and Jenkins methodology.

Moreover, this research specifically seeks to determine if the trend of the monthly average of the quota values for each AFP of the type 2 fund influences the unit root, if stationarity influences its mean and variance, and if there is correlation between the observed values and the forecast values.

General Hypothesis

The monthly average of the quota values for each AFP of the type 2 fund will be adequately modeled with the Box and Jenkins methodology.

Specific Hypotheses

1. The trend of the monthly average of the quota values for each AFP of the type 2 fund directly influences the unit root.
2. Stationarity directly influences the mean of the return of the monthly average of the quota values for each AFP of the type 2 fund.
3. Stationarity directly influences the variance of the return of the monthly average of the quota values for each AFP of the type 2 fund.
4. There is a correlation between the observed values in the monthly average of the quota values for each AFP of the type 2 fund and the predicted values.

METHODOLOGY

Box and Jenkins methodology or ARIMA models were used to describe and forecast the returns of the monthly averages of quota values in soles of the type 2 fund that 4 AFPs—currently in the market—invested from August 2005 to July 2020. The data were extracted from the website of the Superintendencia de Banca, Seguros y AFP[3], section Boletín Estadístico de AFP[4] (Monthly), through the link https://www.sbs.gob.pe/app/stats_net/stats/EstadisticaBoletinEstadistico.aspx?p=31#. The data correspond to the time series. The population for AFP Integra and Profuturo is 180 months from August 2005 to July 2020; for Prima, 179 months from September 2005 to July 2020; and for Habitat, 86 months from June 2013 to July 2020. These data were modeled with the econometric package EVIEWS 10; Stata 16 and Risk simulator—a Monte Carlo simulation software that works as an Excel add-in—were also used. AFP Horizonte and Unión Vida were discarded, since they are not currently in the market, and stationarity was identified as a weakly stationary stochastic process, since the first two moments—the mathematical expectation and the variance of the random variables—are constant and do not depend on time. Moreover, the covariances between two random variables of different periods depend only on the time elapsed between them, a necessary condition for them to be modeled with the Box and Jenkins methodology by means of the following four steps:

Identification

In this part, it was verified, based on the unit root (UR) tests, whether the series of the four AFPs were stationary; in addition, it was verified that the series had memory or that they did not have white noise, since otherwise, they could not be forecast with the Box and Jenkins methodology. For this, the following substeps were performed: graphical analysis, statistics calculations, unit root tests and white noise tests.

Estimation

Based on the results of the correlograms, the order of the AR and MA were identified using maximum

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likelihood estimation and the trial and error method from the statistical significance of each estimated coefficient.

Diagnostic Checking

The unit circle was used to validate the stability of the model, to corroborate that the residuals and squared residuals are white noise, and finally to perform the constant variance test with the following substeps: validation of the unit circle, validation of the residuals, and validation of the squared residuals.

Forecasting

A static forecast *t*+1—that is, one period ahead—was performed to calculate the error statistics. Subsequently, a dynamic forecast *t*+*k* periods was performed.

RESULTS

The results after applying the Box-Jenkins methodology tests, also known as ARIMA, are presented below.

Identification

Graphical Analysis

These are non-formal tests. Court and Rengifo (2011) argue that they help to determine the model and the order that best fit the data, since graphical methods and information criteria are used. Figure 2 shows the development of the monthly series of AFP Habitat, Integra, Prima and Profuturo, which show upward trends.

Statitics Calculation

Figure 2 shows that the original series of AFP Profuturo has a trend, but, in the results of the model in Figure 3 (left side), it has a *p*-value of 4. 95%, that is, less than 5%, so the H_0 (which states that the series has a UR) is rejected and, therefore, it is stationary; however, in the same Figure 3 (right side), it can be observed that the autocorrelation does not decay exponentially to corroborate that the original series is stationary; on the contrary, it decays linearly, which indicates that it is not stationary and must be differentiated.

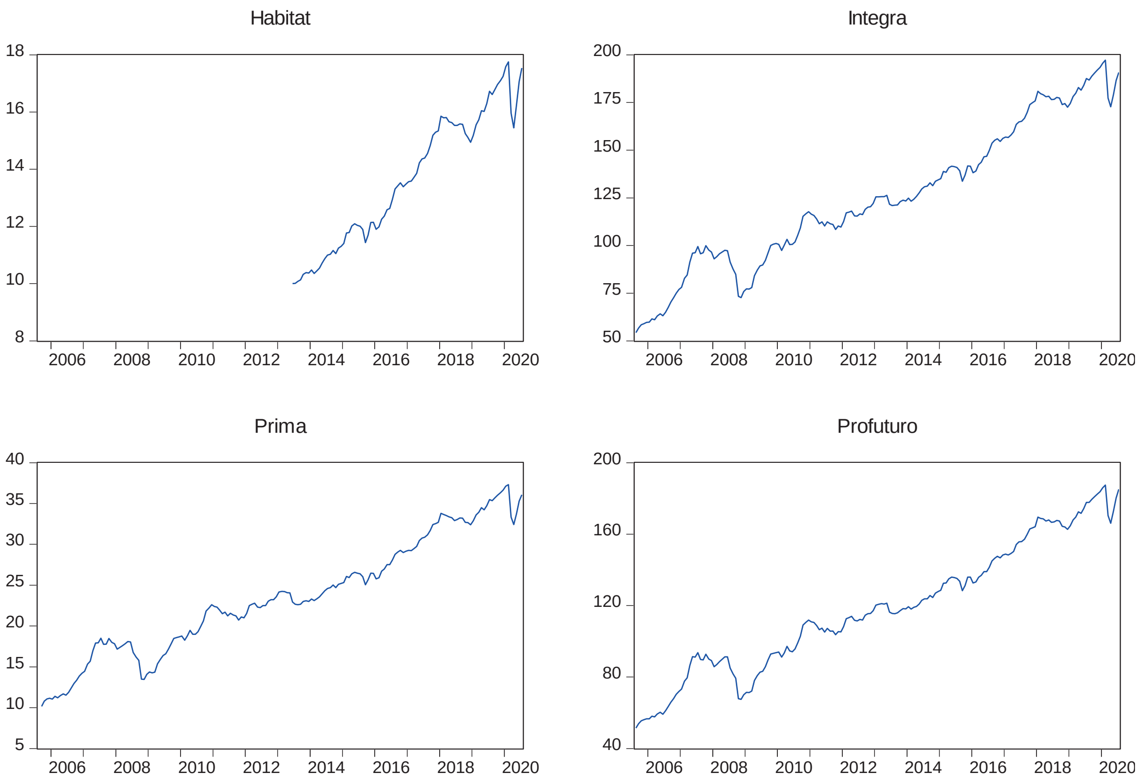

Figure 2. *Behavior of the monthly average series of AFP Habitat, Integra, Prima and Profuturo by levels.*

Source: Prepared by the author using Eviews 10.

Figure 4 shows that the original series of AFP Habitat, Integra and Prima have at least one UR and, therefore, are not stationary. Gujarati and Porter (2010) point out that "Each set of time series data will therefore be for a particular episode. As a consequence, it is not possible to generalize it to other time periods". For forecasting time series, non-stationary time series are not very useful, and to overcome this obstacle, the original series must be differentiated to make them stationary.

Series: PROFUTURO Workfile: ASIMETRIA_FOND...

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Augmented Dickey-Fuller Unit Root Test on PROFUTURO

Null Hypothesis: PROFUTURO has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 1 (Automatic - based on SIC, maxlag=13)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-3.439606	0.0495
Test critical values:	1% level	-4.010440	
	5% level	-3.435269	
	10% level	-3.141649	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(PROFUTURO)
Method: Least Squares
Date: 08/30/20 Time: 13:59
Sample (adjusted): 2005M10 2020M07
Included observations: 178 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
PROFUTURO(-1)	-0.095374	0.027728	-3.439606	0.0007
D(PROFUTURO(-1))	0.313196	0.072409	4.325350	0.0000
C	5.954200	1.635283	3.641082	0.0004
@TREND("2005M08")	0.064529	0.018983	3.399233	0.0008

R-squared	0.129623	Mean dependent var	0.737564
Adjusted R-squared	0.114617	S.D. dependent var	2.741894
S.E. of regression	2.579980	Akaike info criterion	4.755656
Sum squared resid	1158.195	Schwarz criterion	4.827156
Log likelihood	-419.2534	Hannan-Quinn criter.	4.784651
F-statistic	8.637801	Durbin-Watson stat	1.967760
Prob(F-statistic)	0.000022		

Series: PROFUTURO Workfile: ASIMETRIA_FOND...

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Correlogram of PROFUTURO

Date: 08/30/20 Time: 14:30
Sample: 2005M08 2020M07
Included observations: 180

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.977	0.977	174.83	0.000
		2	0.955	-0.007	342.69	0.000
		3	0.935	0.043	504.50	0.000
		4	0.917	0.034	661.03	0.000
		5	0.898	-0.031	811.96	0.000
		6	0.873	-0.130	955.53	0.000
		7	0.849	0.004	1092.2	0.000
		8	0.826	-0.021	1222.1	0.000
		9	0.803	-0.002	1345.7	0.000
		10	0.781	0.012	1463.4	0.000
		11	0.759	-0.001	1575.1	0.000
		12	0.738	0.006	1681.3	0.000
		13	0.717	0.007	1782.2	0.000
		14	0.698	0.019	1878.5	0.000
		15	0.680	0.012	1970.4	0.000
		16	0.662	-0.004	2058.0	0.000
		17	0.645	0.007	2141.7	0.000
		18	0.629	-0.001	2221.7	0.000
		19	0.614	0.029	2298.5	0.000
		20	0.601	0.013	2372.3	0.000
		21	0.589	0.041	2443.9	0.000
		22	0.579	0.030	2513.4	0.000
		23	0.568	-0.027	2580.7	0.000
		24	0.557	0.003	2645.9	0.000

Figure 3. Statistics Calculation for AFP Profuturo.

Source: Prepared by the author using Eviews 10.

Series: HABITAT Workfile: ARIMA_FONDO_T2::Fo...

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Augmented Dickey-Fuller Unit Root Test on HABITAT

Null Hypothesis: HABITAT has a unit root
Exogenous: None
Lag Length: 2 (Automatic - based on SIC, maxlag=11)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		2.665431	0.9980
Test critical values:	1% level	-2.593121	
	5% level	-1.944762	
	10% level	-1.614204	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(HABITAT)
Method: Least Squares
Date: 09/17/20 Time: 18:57
Sample (adjusted): 2013M09 2020M07
Included observations: 83 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
HABITAT(-1)	0.006245	0.002343	2.665431	0.0093
D(HABITAT(-1))	0.324699	0.105306	3.083386	0.0028
D(HABITAT(-2))	-0.371749	0.109312	-3.400802	0.0011

R-squared	0.172645	Mean dependent var	0.089810
Adjusted R-squared	0.151962	S.D. dependent var	0.301719
S.E. of regression	0.277850	Akaike info criterion	0.312007
Sum squared resid	6.176067	Schwarz criterion	0.399435
Log likelihood	-9.948305	Hannan-Quinn criter.	0.347131
Durbin-Watson stat	2.035746		

Series: INTEGRA Workfile: ASIMETRIA_FONDO_T...

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Augmented Dickey-Fuller Unit Root Test on INTEGRA

Null Hypothesis: INTEGRA has a unit root
Exogenous: None
Lag Length: 1 (Automatic - based on SIC, maxlag=13)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		2.256207	0.9944
Test critical values:	1% level	-2.578018	
	5% level	-1.942624	
	10% level	-1.615515	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(INTEGRA)
Method: Least Squares
Date: 08/30/20 Time: 13:49
Sample (adjusted): 2005M10 2020M07
Included observations: 178 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
INTEGRA(-1)	0.003800	0.001684	2.256207	0.0253
D(INTEGRA(-1))	0.259096	0.073383	3.530734	0.0005

R-squared	0.054761	Mean dependent var	0.753146
Adjusted R-squared	0.049390	S.D. dependent var	2.911200
S.E. of regression	2.838397	Akaike info criterion	4.935528
Sum squared resid	1417.944	Schwarz criterion	4.971279
Log likelihood	-437.2620	Hannan-Quinn criter.	4.950026
Durbin-Watson stat	1.951355		

Series: PRIMA Workfile: ASIMETRIA_FONDO_T2::...

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Augmented Dickey-Fuller Unit Root Test on PRIMA

Null Hypothesis: PRIMA has a unit root
Exogenous: None
Lag Length: 1 (Automatic - based on SIC, maxlag=13)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		2.155587	0.9927
Test critical values:	1% level	-2.578092	
	5% level	-1.942634	
	10% level	-1.615508	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(PRIMA)
Method: Least Squares
Date: 08/30/20 Time: 13:55
Sample (adjusted): 2005M11 2020M07
Included observations: 177 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
PRIMA(-1)	0.003781	0.001754	2.155587	0.0325
D(PRIMA(-1))	0.263942	0.073341	3.598828	0.0004

R-squared	0.058431	Mean dependent var	0.142771
Adjusted R-squared	0.053051	S.D. dependent var	0.571698
S.E. of regression	0.556327	Akaike info criterion	1.676315
Sum squared resid	54.16251	Schwarz criterion	1.712204
Log likelihood	-146.3539	Hannan-Quinn criter.	1.690870
Durbin-Watson stat	1.939383		

Figure 4. Statistics Calculation for AFP Integra, Prima and Profuturo.

Source: Prepared by the author using Eviews 10.

Unit Root Tests

Dickey Fuller Augmented (DFA) tests, which according to Bello (2018) are the test most widely used, were performed on the original series of Habitat, Integra, Prima and Profuturo.

When performing the hypothesis tests to those series, the null hypotheses that suggested that the series have at least one UR were not rejected, therefore, the logarithmic differentiation of their original series was applied to make them stationary. Figure 5 shows the results of the models.

White Noise Tests

It was verified that the time series already differentiated had memory using correlograms and the Ljung Box (LB) statistic for small samples. Figure 6 shows the correlograms for Habitat, which showed that up to month seven there is no white noise and, from month eight, the impact on the current Habitat series is not significant. In the case of Integra, there is no white noise up to month fifteen. As for Prima, there is no white noise up to month thirteen. Finally, Profuturo has no white noise until month sixteen and, after this month, the impact is not significant

Series: DLOGHABITAT Workfile: ARIMA_FONDO_...

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Augmented Dickey-Fuller Unit Root Test on DLOGHABITAT

Null Hypothesis: DLOGHABITAT has a unit root
Exogenous: Constant
Lag Length: 1 (Automatic - based on SIC, maxlag=11)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-7.712343	0.0000
Test critical values:	1% level	-3.511262	
	5% level	-2.896779	
	10% level	-2.585626	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(DLOGHABITAT)
Method: Least Squares
Date: 09/17/20 Time: 23:57
Sample (adjusted): 2013M09 2020M07
Included observations: 83 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
DLOGHABITAT(-1)	-1.043541	0.135308	-7.712343	0.0000
D(DLOGHABITAT(-1))	0.340585	0.108978	3.125258	0.0025
C	0.006754	0.002170	3.112476	0.0026

R-squared	0.449351	Mean dependent var	0.000249
Adjusted R-squared	0.435585	S.D. dependent var	0.024314
S.E. of regression	0.018267	Akaike info criterion	-5.131990
Sum squared resid	0.026694	Schwarz criterion	-5.044562
Log likelihood	215.9776	Hannan-Quinn criter.	-5.096866
F-statistic	32.64161	Durbin-Watson stat	2.026463
Prob(F-statistic)	0.000000		

Series: DLOGINTEGRA Workfile: ARIMA_FONDO...

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Augmented Dickey-Fuller Unit Root Test on DLOGINTEGRA

Null Hypothesis: DLOGINTEGRA has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=13)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-9.711752	0.0000
Test critical values:	1% level	-3.467205	
	5% level	-2.877636	
	10% level	-2.575430	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(DLOGINTEGRA)
Method: Least Squares
Date: 09/18/20 Time: 17:54
Sample (adjusted): 2005M10 2020M07
Included observations: 178 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
DLOGINTEGRA(-1)	-0.692758	0.071332	-9.711752	0.0000
C	0.004683	0.001863	2.513300	0.0129

R-squared	0.348915	Mean dependent var	-0.000120
Adjusted R-squared	0.345216	S.D. dependent var	0.029618
S.E. of regression	0.023966	Akaike info criterion	-4.613166
Sum squared resid	0.101091	Schwarz criterion	-4.577415
Log likelihood	412.5717	Hannan-Quinn criter.	-4.598668
F-statistic	94.31812	Durbin-Watson stat	2.007516
Prob(F-statistic)	0.000000		

Series: DLOGPRIMA Workfile: ARIMA_FONDO_T2...

View | Proc | Object | Properties | Print | Name | Freeze | Sample | Genr | Sheet | Graph | Sta

Augmented Dickey-Fuller Unit Root Test on DLOGPRIMA

Null Hypothesis: DLOGPRIMA has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=13)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-9.954895	0.0000
Test critical values:	1% level	-3.467418	
	5% level	-2.877729	
	10% level	-2.575480	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(DLOGPRIMA)
Method: Least Squares
Date: 09/18/20 Time: 18:01
Sample (adjusted): 2005M11 2020M07
Included observations: 177 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
DLOGPRIMA(-1)	-0.711947	0.071517	-9.954895	0.0000
C	0.004788	0.001954	2.450531	0.0152

R-squared	0.361547	Mean dependent var	-0.000222
Adjusted R-squared	0.357898	S.D. dependent var	0.031343
S.E. of regression	0.025115	Akaike info criterion	-4.519436
Sum squared resid	0.110387	Schwarz criterion	-4.483547
Log likelihood	401.9701	Hannan-Quinn criter.	-4.504881
F-statistic	99.09993	Durbin-Watson stat	2.000463
Prob(F-statistic)	0.000000		

Series: DLOGPROFUTURO Workfile: ARIMA_FOND...

View | Proc | Object | Properties | Print | Name | Freeze | Sample | Genr | Sheet | Graph | Stat

Augmented Dickey-Fuller Unit Root Test on DLOGPROFUTURO

Null Hypothesis: DLOGPROFUTURO has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=13)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-9.707698	0.0000
Test critical values:	1% level	-3.467205	
	5% level	-2.877636	
	10% level	-2.575430	

*MacKinnon (1996) one-sided p-values.

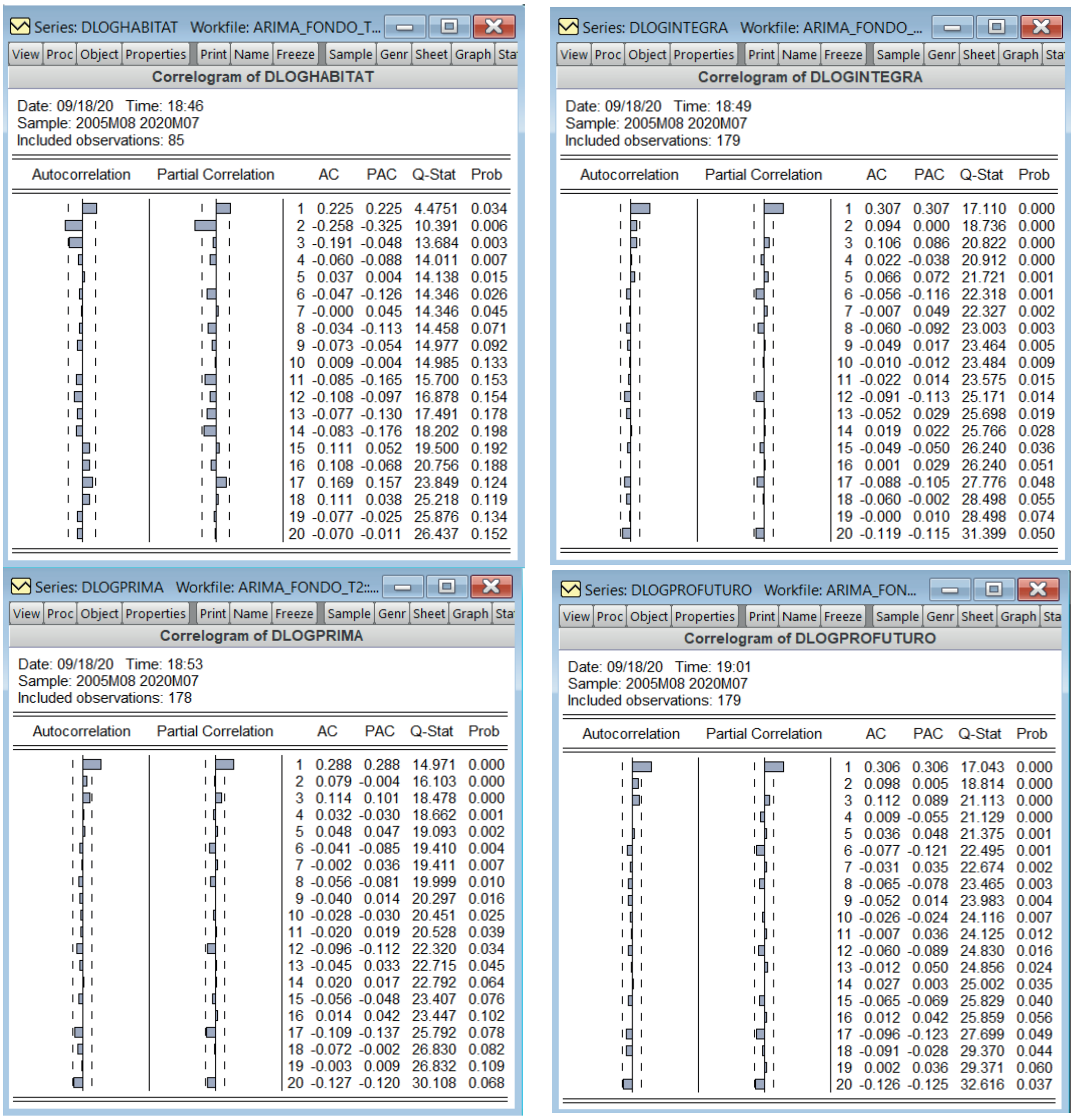
Augmented Dickey-Fuller Test Equation
Dependent Variable: D(DLOGPROFUTURO)
Method: Least Squares
Date: 09/18/20 Time: 18:06
Sample (adjusted): 2005M10 2020M07
Included observations: 178 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
DLOGPROFUTURO(...	-0.692971	0.071384	-9.707698	0.0000
C	0.004776	0.001902	2.510810	0.0129

R-squared	0.348726	Mean dependent var	-0.000100
Adjusted R-squared	0.345025	S.D. dependent var	0.030247
S.E. of regression	0.024479	Akaike info criterion	-4.570831
Sum squared resid	0.105463	Schwarz criterion	-4.535081
Log likelihood	408.8040	Hannan-Quinn criter.	-4.556333
F-statistic	94.23940	Durbin-Watson stat	2.010274
Prob(F-statistic)	0.000000		

Figure 5. Unit root tests.

Source: Prepared by the author using Eviews 10.



Series: DLOGHABITAT Workfile: ARIMA_FONDO_T...

Correlogram of DLOGHABITAT

Date: 09/18/20 Time: 18:46
Sample: 2005M08 2020M07
Included observations: 85

	AC	PAC	Q-Stat	Prob
1	0.225	0.225	4.4751	0.034
2	-0.258	-0.325	10.391	0.006
3	-0.191	-0.048	13.684	0.003
4	-0.060	-0.088	14.011	0.007
5	0.037	0.004	14.138	0.015
6	-0.047	-0.126	14.346	0.026
7	-0.000	0.045	14.346	0.045
8	-0.034	-0.113	14.458	0.071
9	-0.073	-0.054	14.977	0.092
10	0.009	-0.004	14.985	0.133
11	-0.085	-0.165	15.700	0.153
12	-0.108	-0.097	16.878	0.154
13	-0.077	-0.130	17.491	0.178
14	-0.083	-0.176	18.202	0.198
15	0.111	0.052	19.500	0.192
16	0.108	-0.068	20.756	0.188
17	0.169	0.157	23.849	0.124
18	0.111	0.038	25.218	0.119
19	-0.077	-0.025	25.876	0.134
20	-0.070	-0.011	26.437	0.152

Series: DLOGINTEGRA Workfile: ARIMA_FONDO_...

Correlogram of DLOGINTEGRA

Date: 09/18/20 Time: 18:49
Sample: 2005M08 2020M07
Included observations: 179

	AC	PAC	Q-Stat	Prob
1	0.307	0.307	17.110	0.000
2	0.094	0.000	18.736	0.000
3	0.106	0.086	20.822	0.000
4	0.022	-0.038	20.912	0.000
5	0.066	0.072	21.721	0.001
6	-0.056	-0.116	22.318	0.001
7	-0.007	0.049	22.327	0.002
8	-0.060	-0.092	23.003	0.003
9	-0.049	0.017	23.464	0.005
10	-0.010	-0.012	23.484	0.009
11	-0.022	0.014	23.575	0.015
12	-0.091	-0.113	25.171	0.014
13	-0.052	0.029	25.698	0.019
14	0.019	0.022	25.766	0.028
15	-0.049	-0.050	26.240	0.036
16	0.001	0.029	26.240	0.051
17	-0.088	-0.105	27.776	0.048
18	-0.060	-0.002	28.498	0.055
19	-0.000	0.010	28.498	0.074
20	-0.119	-0.115	31.399	0.050

Series: DLOGPRIMA Workfile: ARIMA_FONDO_T2:...

Correlogram of DLOGPRIMA

Date: 09/18/20 Time: 18:53
Sample: 2005M08 2020M07
Included observations: 178

	AC	PAC	Q-Stat	Prob
1	0.288	0.288	14.971	0.000
2	0.079	-0.004	16.103	0.000
3	0.114	0.101	18.478	0.000
4	0.032	-0.030	18.662	0.001
5	0.048	0.047	19.093	0.002
6	-0.041	-0.085	19.410	0.004
7	-0.002	0.036	19.411	0.007
8	-0.056	-0.081	19.999	0.010
9	-0.040	0.014	20.297	0.016
10	-0.028	-0.030	20.451	0.025
11	-0.020	0.019	20.528	0.039
12	-0.096	-0.112	22.320	0.034
13	-0.045	0.033	22.715	0.045
14	0.020	0.017	22.792	0.064
15	-0.056	-0.048	23.407	0.076
16	0.014	0.042	23.447	0.102
17	-0.109	-0.137	25.792	0.078
18	-0.072	-0.002	26.830	0.082
19	-0.003	0.009	26.832	0.109
20	-0.127	-0.120	30.108	0.068

Series: DLOGPROFUTURO Workfile: ARIMA_FON...

Correlogram of DLOGPROFUTURO

Date: 09/18/20 Time: 19:01
Sample: 2005M08 2020M07
Included observations: 179

	AC	PAC	Q-Stat	Prob
1	0.306	0.306	17.043	0.000
2	0.098	0.005	18.814	0.000
3	0.112	0.089	21.113	0.000
4	0.009	-0.055	21.129	0.000
5	0.036	0.048	21.375	0.001
6	-0.077	-0.121	22.495	0.001
7	-0.031	0.035	22.674	0.002
8	-0.065	-0.078	23.465	0.003
9	-0.052	0.014	23.983	0.004
10	-0.026	-0.024	24.116	0.007
11	-0.007	0.036	24.125	0.012
12	-0.060	-0.089	24.830	0.016
13	-0.012	0.050	24.856	0.024
14	0.027	0.003	25.002	0.035
15	-0.065	-0.069	25.829	0.040
16	0.012	0.042	25.859	0.056
17	-0.096	-0.123	27.699	0.049
18	-0.091	-0.028	29.370	0.044
19	0.002	0.036	29.371	0.060
20	-0.126	-0.125	32.616	0.037

Figure *6.* White noise tests.

Source: Prepared by the author using Eviews 10.

in the result of the current series. In conclusion, the differentiated series of AFP Habitat, Integra, Prima and Profuturo are stationary and have no white noise and, therefore, can be forecast with the Box and Jenkins methodology.

Estimation

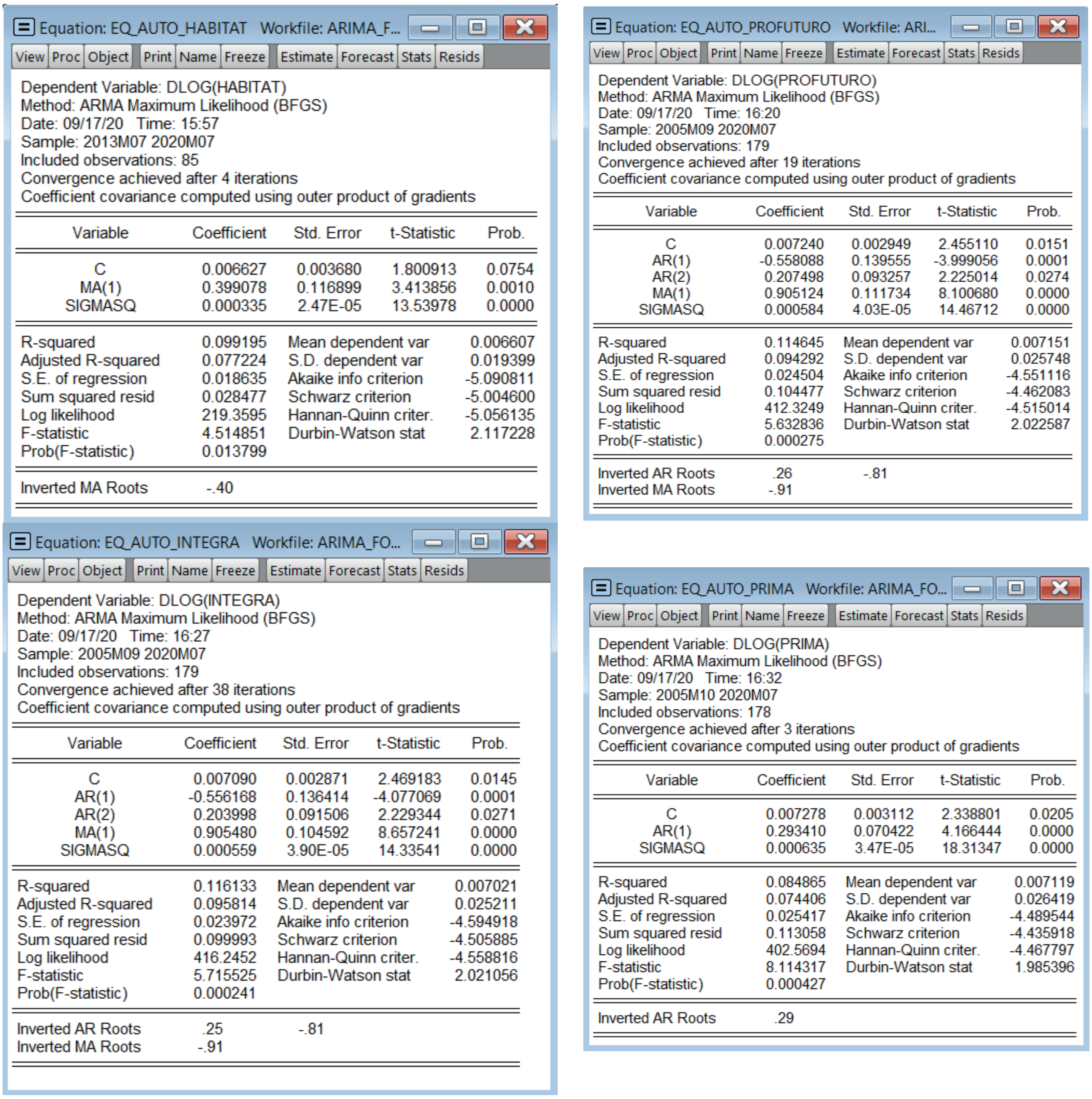
Based on the results of the correlograms, the order of the AR and MA were identified using maximum likelihood estimation, and the trial and error method, based on the statistical significance of each estimated coefficient. Using Eviews software, the best model was automatically selected by running iterations with combinations of the AR, MA and order of integration. In this case, 484 models were run for each AFP, and the model with the lowest Akaike Info Criterion (AIC) was chosen, as shown in Figure 7.

The representations of this model are shown in Table 1.

Validation

Unit Circle Validation

The validation with the unit circle was performed and, as shown in Figure 8, it can be seen that the models are stationary in the autoregressive part for AFP Prima, Profuturo and Integra. It is also observed that the models of AFP Habitat, Profuturo and Integra are invertible in the moving average



Equation: EQ_AUTO_HABITAT Workfile: ARIMA_F...

View | Proc | Object | Print | Name | Freeze | Estimate | Forecast | Stats | Resids

Dependent Variable: DLOG(HABITAT)
Method: ARMA Maximum Likelihood (BFGS)
Date: 09/17/20 Time: 15:57
Sample: 2013M07 2020M07
Included observations: 85
Convergence achieved after 4 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.006627	0.003680	1.800913	0.0754
MA(1)	0.399078	0.116899	3.413856	0.0010
SIGMASQ	0.000335	2.47E-05	13.53978	0.0000

R-squared	0.099195	Mean dependent var	0.006607
Adjusted R-squared	0.077224	S.D. dependent var	0.019399
S.E. of regression	0.018635	Akaike info criterion	-5.090811
Sum squared resid	0.028477	Schwarz criterion	-5.004600
Log likelihood	219.3595	Hannan-Quinn criter.	-5.056135
F-statistic	4.514851	Durbin-Watson stat	2.117228
Prob(F-statistic)	0.013799		

Inverted MA Roots -.40

Equation: EQ_AUTO_PROFUTURO Workfile: ARI...

View | Proc | Object | Print | Name | Freeze | Estimate | Forecast | Stats | Resids

Dependent Variable: DLOG(PROFUTURO)
Method: ARMA Maximum Likelihood (BFGS)
Date: 09/17/20 Time: 16:20
Sample: 2005M09 2020M07
Included observations: 179
Convergence achieved after 19 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.007240	0.002949	2.455110	0.0151
AR(1)	-0.558088	0.139555	-3.999056	0.0001
AR(2)	0.207498	0.093257	2.225014	0.0274
MA(1)	0.905124	0.111734	8.100680	0.0000
SIGMASQ	0.000584	4.03E-05	14.46712	0.0000

R-squared	0.114645	Mean dependent var	0.007151
Adjusted R-squared	0.094292	S.D. dependent var	0.025748
S.E. of regression	0.024504	Akaike info criterion	-4.551116
Sum squared resid	0.104477	Schwarz criterion	-4.462083
Log likelihood	412.3249	Hannan-Quinn criter.	-4.515014
F-statistic	5.632836	Durbin-Watson stat	2.022587
Prob(F-statistic)	0.000275		

Inverted AR Roots .26 -.81
Inverted MA Roots -.91

Equation: EQ_AUTO_INTEGRA Workfile: ARIMA_FO...

View | Proc | Object | Print | Name | Freeze | Estimate | Forecast | Stats | Resids

Dependent Variable: DLOG(INTEGRA)
Method: ARMA Maximum Likelihood (BFGS)
Date: 09/17/20 Time: 16:27
Sample: 2005M09 2020M07
Included observations: 179
Convergence achieved after 38 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.007090	0.002871	2.469183	0.0145
AR(1)	-0.556168	0.136414	-4.077069	0.0001
AR(2)	0.203998	0.091506	2.229344	0.0271
MA(1)	0.905480	0.104592	8.657241	0.0000
SIGMASQ	0.000559	3.90E-05	14.33541	0.0000

R-squared	0.116133	Mean dependent var	0.007021
Adjusted R-squared	0.095814	S.D. dependent var	0.025211
S.E. of regression	0.023972	Akaike info criterion	-4.594918
Sum squared resid	0.099993	Schwarz criterion	-4.505885
Log likelihood	416.2452	Hannan-Quinn criter.	-4.558816
F-statistic	5.715525	Durbin-Watson stat	2.021056
Prob(F-statistic)	0.000241		

Inverted AR Roots .25 -.81
Inverted MA Roots -.91

Equation: EQ_AUTO_PRIMA Workfile: ARIMA_FO...

View | Proc | Object | Print | Name | Freeze | Estimate | Forecast | Stats | Resids

Dependent Variable: DLOG(PRIMA)
Method: ARMA Maximum Likelihood (BFGS)
Date: 09/17/20 Time: 16:32
Sample: 2005M10 2020M07
Included observations: 178
Convergence achieved after 3 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.007278	0.003112	2.338801	0.0205
AR(1)	0.293410	0.070422	4.166444	0.0000
SIGMASQ	0.000635	3.47E-05	18.31347	0.0000

R-squared	0.084865	Mean dependent var	0.007119
Adjusted R-squared	0.074406	S.D. dependent var	0.026419
S.E. of regression	0.025417	Akaike info criterion	-4.489544
Sum squared resid	0.113058	Schwarz criterion	-4.435918
Log likelihood	402.5694	Hannan-Quinn criter.	-4.467797
F-statistic	8.114317	Durbin-Watson stat	1.985396
Prob(F-statistic)	0.000427		

Inverted AR Roots .29

Figure 7. Identification of the order of AR and MA.

Source: Prepared by the author using Eviews 10.

Table 1. *Selection of ARIMA Models.*

AFP	Representation
Habitat	ARIMA (0,1,1)
Profuturo	ARIMA (2,1,1)
Integra	ARIMA (2,1,1)
Prima	ARIMA (1,1,0)

Source: Prepared by the author.

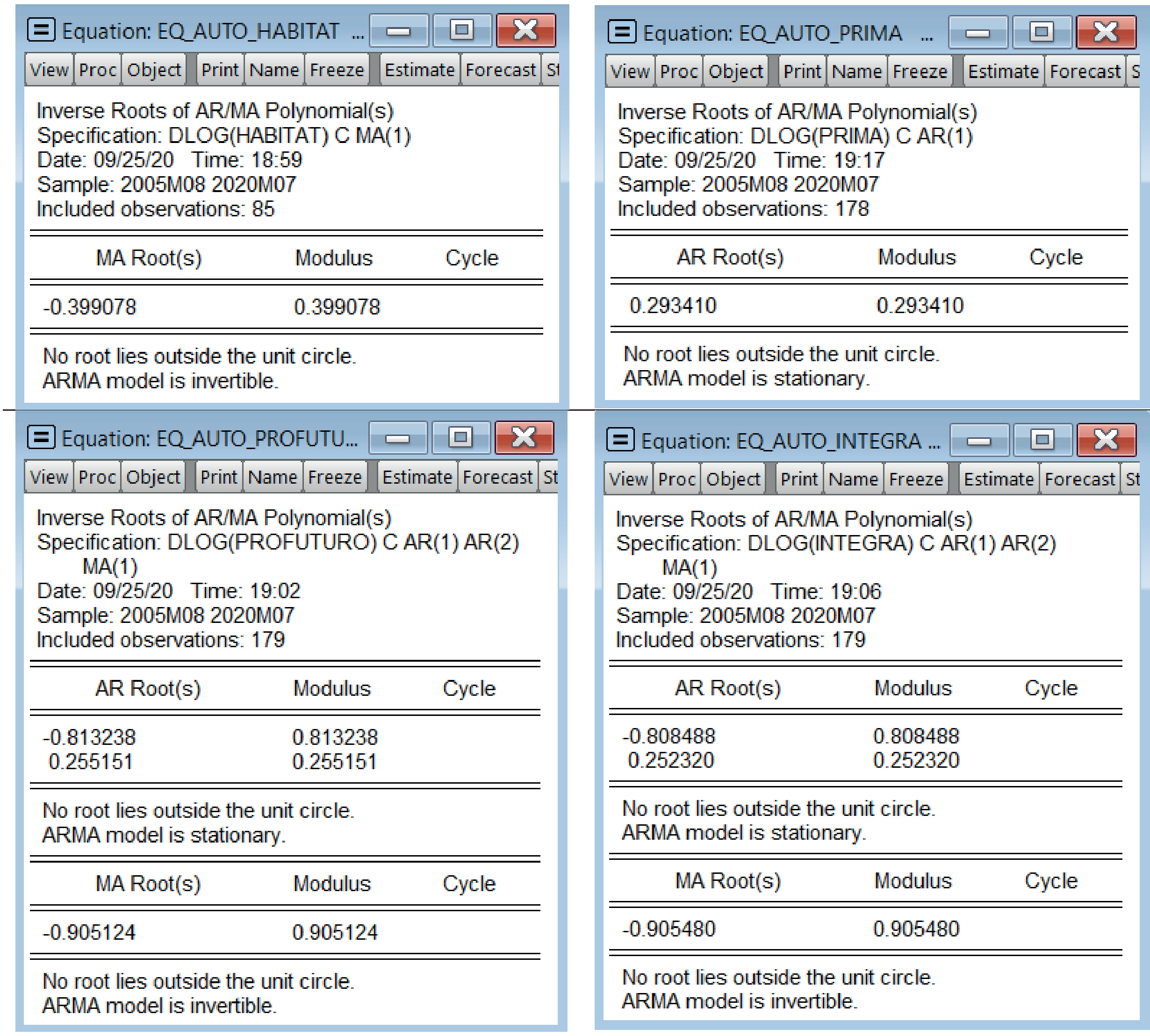


Figure 8. Unit circle validation.

Source: Prepared by the author using Eviews 10.

part. None of the roots of the four AFPs are outside the unit circle, and all their moduli are below 1, so it is concluded that they passed the unit circle validation tests.

Validation of the residuals

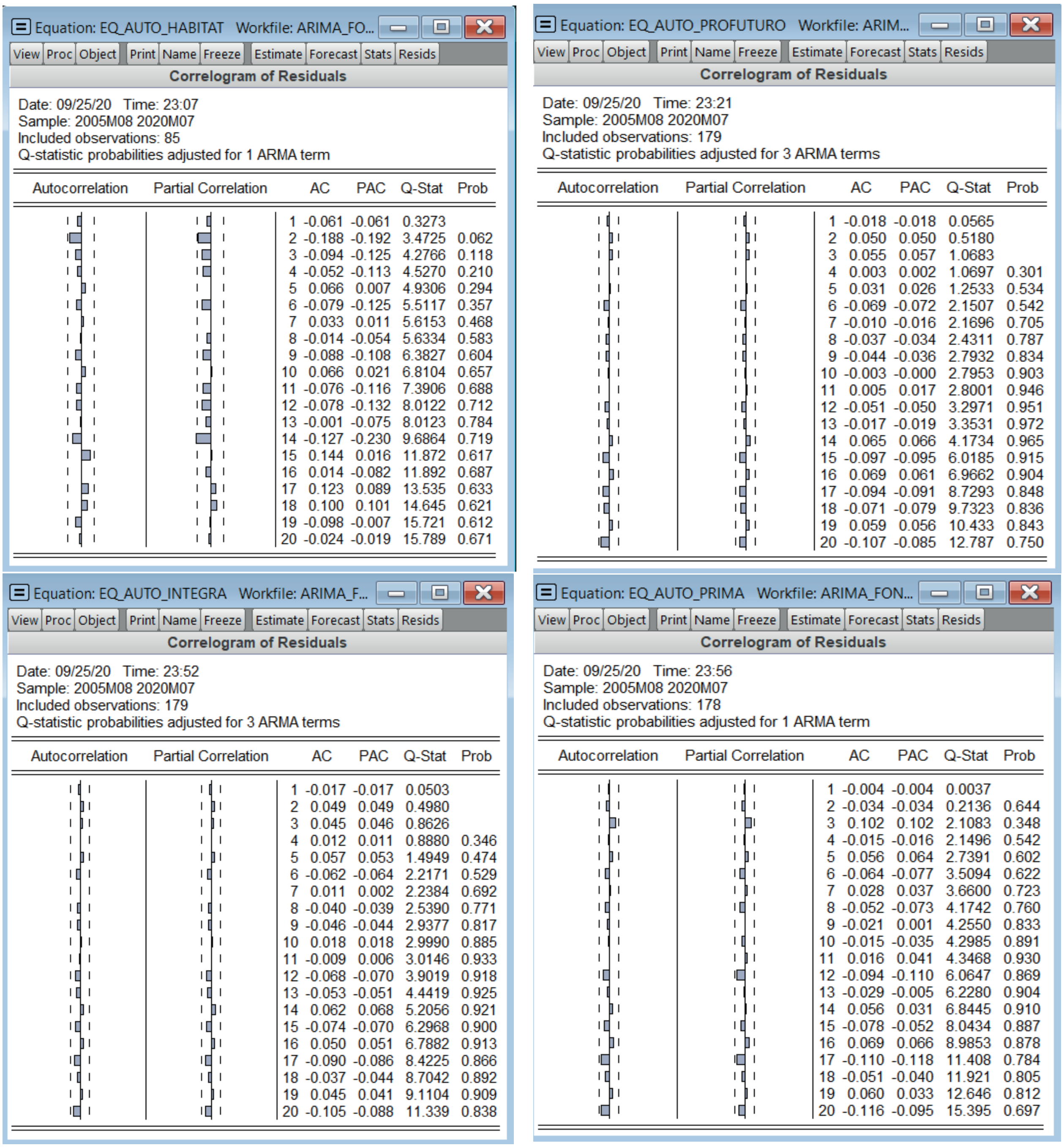
Subsequently, the correlogram was performed to verify that the residuals had white noise. In Figure 9, it was verified that the behavior of the residuals of the four AFPs has white noise because their *p*-values are greater than 5% and, therefore, forecasting with the Box and Jenkins ARIMA models is possible.

Validation of the squared residuals

The squared residuals of the 4 AFPs also presented white noise, since all their *p*-values are above 5%, as shown in Figure 10; if this had not been the case, it would have been necessary to perform an equation to the variable and then work with the conditional volatility models ARCH and GARCH.

Forecasting

Curt and Rengifo (2011) argue that *Para determinar si un pronóstico es adecuado, se usan los estadísticos que (…) comparan los valores reales con aquellos que han sido pronosticados. (…) como los errores pueden ser positivos o negativos, (…) suma de ellos no sería de gran ayuda puesto que se cancelarían entre ellos. Es por eso que los índices trabajan ya sea con los errores al cuadrado o con el valor absoluto de los errores* [To determine whether a forecast is adequate, statistics that (...)



Equation: EQ_AUTO_HABITAT Workfile: ARIMA_FO...

View | Proc | Object | Print | Name | Freeze | Estimate | Forecast | Stats | Resids

Correlogram of Residuals

Date: 09/25/20 Time: 23:07
Sample: 2005M08 2020M07
Included observations: 85
Q-statistic probabilities adjusted for 1 ARMA term

	AC	PAC	Q-Stat	Prob
1	-0.061	-0.061	0.3273	
2	-0.188	-0.192	3.4725	0.062
3	-0.094	-0.125	4.2766	0.118
4	-0.052	-0.113	4.5270	0.210
5	0.066	0.007	4.9306	0.294
6	-0.079	-0.125	5.5117	0.357
7	0.033	0.011	5.6153	0.468
8	-0.014	-0.054	5.6334	0.583
9	-0.088	-0.108	6.3827	0.604
10	0.066	0.021	6.8104	0.657
11	-0.076	-0.116	7.3906	0.688
12	-0.078	-0.132	8.0122	0.712
13	-0.001	-0.075	8.0123	0.784
14	-0.127	-0.230	9.6864	0.719
15	0.144	0.016	11.872	0.617
16	0.014	-0.082	11.892	0.687
17	0.123	0.089	13.535	0.633
18	0.100	0.101	14.645	0.621
19	-0.098	-0.007	15.721	0.612
20	-0.024	-0.019	15.789	0.671

Equation: EQ_AUTO_PROFUTURO Workfile: ARIM...

View | Proc | Object | Print | Name | Freeze | Estimate | Forecast | Stats | Resids

Correlogram of Residuals

Date: 09/25/20 Time: 23:21
Sample: 2005M08 2020M07
Included observations: 179
Q-statistic probabilities adjusted for 3 ARMA terms

	AC	PAC	Q-Stat	Prob
1	-0.018	-0.018	0.0565	
2	0.050	0.050	0.5180	
3	0.055	0.057	1.0683	
4	0.003	0.002	1.0697	0.301
5	0.031	0.026	1.2533	0.534
6	-0.069	-0.072	2.1507	0.542
7	-0.010	-0.016	2.1696	0.705
8	-0.037	-0.034	2.4311	0.787
9	-0.044	-0.036	2.7932	0.834
10	-0.003	-0.000	2.7953	0.903
11	0.005	0.017	2.8001	0.946
12	-0.051	-0.050	3.2971	0.951
13	-0.017	-0.019	3.3531	0.972
14	0.065	0.066	4.1734	0.965
15	-0.097	-0.095	6.0185	0.915
16	0.069	0.061	6.9662	0.904
17	-0.094	-0.091	8.7293	0.848
18	-0.071	-0.079	9.7323	0.836
19	0.059	0.056	10.433	0.843
20	-0.107	-0.085	12.787	0.750

Equation: EQ_AUTO_INTEGRA Workfile: ARIMA_F...

View | Proc | Object | Print | Name | Freeze | Estimate | Forecast | Stats | Resids

Correlogram of Residuals

Date: 09/25/20 Time: 23:52
Sample: 2005M08 2020M07
Included observations: 179
Q-statistic probabilities adjusted for 3 ARMA terms

	AC	PAC	Q-Stat	Prob
1	-0.017	-0.017	0.0503	
2	0.049	0.049	0.4980	
3	0.045	0.046	0.8626	
4	0.012	0.011	0.8880	0.346
5	0.057	0.053	1.4949	0.474
6	-0.062	-0.064	2.2171	0.529
7	0.011	0.002	2.2384	0.692
8	-0.040	-0.039	2.5390	0.771
9	-0.046	-0.044	2.9377	0.817
10	0.018	0.018	2.9990	0.885
11	-0.009	0.006	3.0146	0.933
12	-0.068	-0.070	3.9019	0.918
13	-0.053	-0.051	4.4419	0.925
14	0.062	0.068	5.2056	0.921
15	-0.074	-0.070	6.2968	0.900
16	0.050	0.051	6.7882	0.913
17	-0.090	-0.086	8.4225	0.866
18	-0.037	-0.044	8.7042	0.892
19	0.045	0.041	9.1104	0.909
20	-0.105	-0.088	11.339	0.838

Equation: EQ_AUTO_PRIMA Workfile: ARIMA_FON...

View | Proc | Object | Print | Name | Freeze | Estimate | Forecast | Stats | Resids

Correlogram of Residuals

Date: 09/25/20 Time: 23:56
Sample: 2005M08 2020M07
Included observations: 178
Q-statistic probabilities adjusted for 1 ARMA term

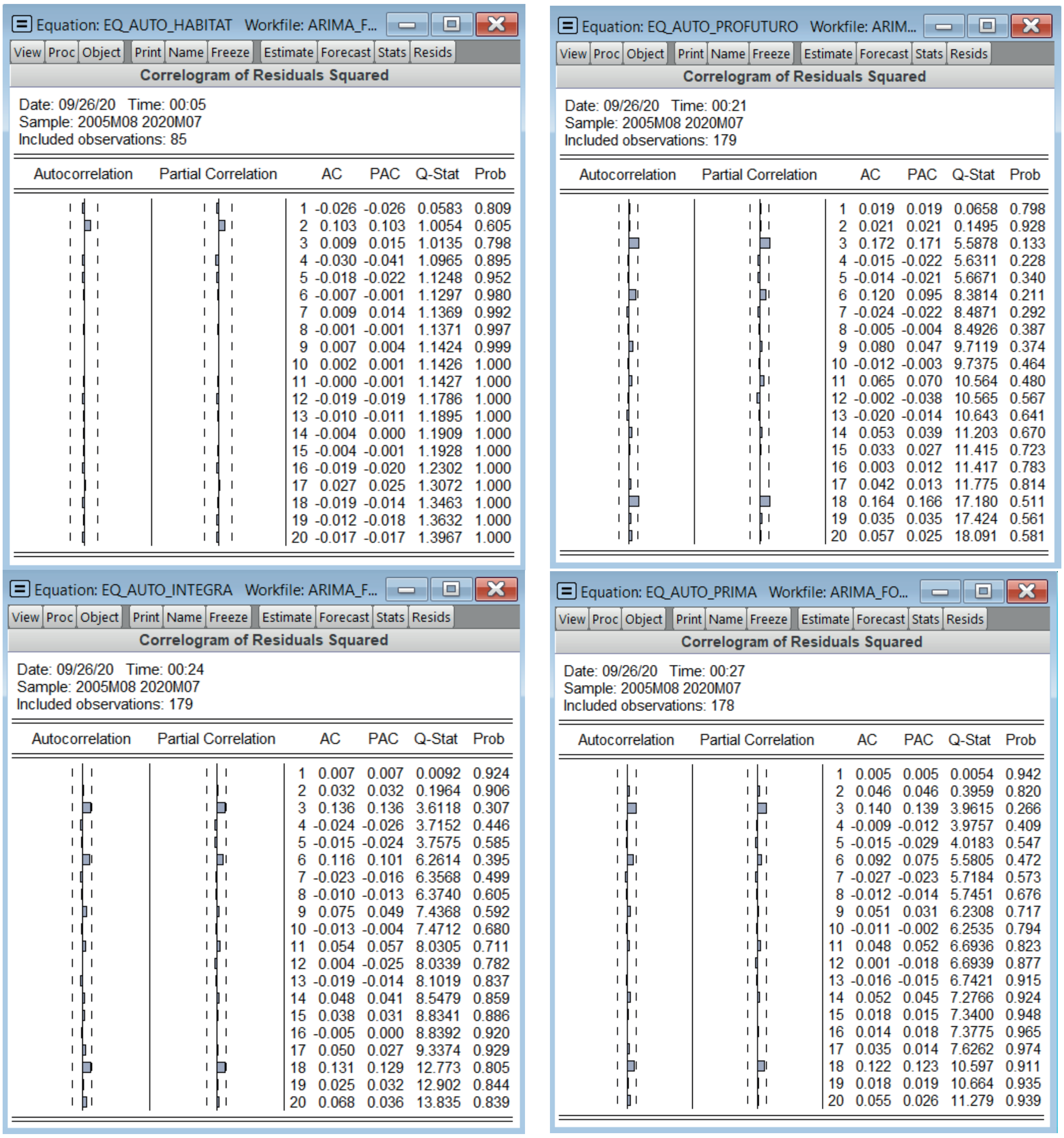
	AC	PAC	Q-Stat	Prob
1	-0.004	-0.004	0.0037	
2	-0.034	-0.034	0.2136	0.644
3	0.102	0.102	2.1083	0.348
4	-0.015	-0.016	2.1496	0.542
5	0.056	0.064	2.7391	0.602
6	-0.064	-0.077	3.5094	0.622
7	0.028	0.037	3.6600	0.723
8	-0.052	-0.073	4.1742	0.760
9	-0.021	0.001	4.2550	0.833
10	-0.015	-0.035	4.2985	0.891
11	0.016	0.041	4.3468	0.930
12	-0.094	-0.110	6.0647	0.869
13	-0.029	-0.005	6.2280	0.904
14	0.056	0.031	6.8445	0.910
15	-0.078	-0.052	8.0434	0.887
16	0.069	0.066	8.9853	0.878
17	-0.110	-0.118	11.408	0.784
18	-0.051	-0.040	11.921	0.805
19	0.060	0.033	12.646	0.812
20	-0.116	-0.095	15.395	0.697

Figure 9. Validation of the residuals.

Source: Prepared by the author using Eviews 10.

compare actual values with those that have been predicted are used. (...) as errors can be positive or negative, (...) to use addition with them would not be of great help since they would cancel each other. That is why the indices work either with squared errors or with the absolute value of the errors] (pp. 427-428); so static forecasts of the monthly average of the quota values of each of the four AFPs are performed in order to take into account the following error statistics: **RMSE** (Root Mean Square Error), **MAE** (Mean Absolute Error), **MAPE** (Mean Absolute Percentage Error) and **U-THEIL** (Theil's Inequality Coefficient), which are contained in Table 2.

Table 2 shows that AFP Habitat has the lowest forecast error with ARIMA, since its RMSE, on average,



Equation: EQ_AUTO_HABITAT Workfile: ARIMA_F...

Correlogram of Residuals Squared

Date: 09/26/20 Time: 00:05
Sample: 2005M08 2020M07
Included observations: 85

	AC	PAC	Q-Stat	Prob
1	-0.026	-0.026	0.0583	0.809
2	0.103	0.103	1.0054	0.605
3	0.009	0.015	1.0135	0.798
4	-0.030	-0.041	1.0965	0.895
5	-0.018	-0.022	1.1248	0.952
6	-0.007	-0.001	1.1297	0.980
7	0.009	0.014	1.1369	0.992
8	-0.001	-0.001	1.1371	0.997
9	0.007	0.004	1.1424	0.999
10	0.002	0.001	1.1426	1.000
11	-0.000	-0.001	1.1427	1.000
12	-0.019	-0.019	1.1786	1.000
13	-0.010	-0.011	1.1895	1.000
14	-0.004	0.000	1.1909	1.000
15	-0.004	-0.001	1.1928	1.000
16	-0.019	-0.020	1.2302	1.000
17	0.027	0.025	1.3072	1.000
18	-0.019	-0.014	1.3463	1.000
19	-0.012	-0.018	1.3632	1.000
20	-0.017	-0.017	1.3967	1.000

Equation: EQ_AUTO_PROFUTURO Workfile: ARIM...

Correlogram of Residuals Squared

Date: 09/26/20 Time: 00:21
Sample: 2005M08 2020M07
Included observations: 179

	AC	PAC	Q-Stat	Prob
1	0.019	0.019	0.0658	0.798
2	0.021	0.021	0.1495	0.928
3	0.172	0.171	5.5878	0.133
4	-0.015	-0.022	5.6311	0.228
5	-0.014	-0.021	5.6671	0.340
6	0.120	0.095	8.3814	0.211
7	-0.024	-0.022	8.4871	0.292
8	-0.005	-0.004	8.4926	0.387
9	0.080	0.047	9.7119	0.374
10	-0.012	-0.003	9.7375	0.464
11	0.065	0.070	10.564	0.480
12	-0.002	-0.038	10.565	0.567
13	-0.020	-0.014	10.643	0.641
14	0.053	0.039	11.203	0.670
15	0.033	0.027	11.415	0.723
16	0.003	0.012	11.417	0.783
17	0.042	0.013	11.775	0.814
18	0.164	0.166	17.180	0.511
19	0.035	0.035	17.424	0.561
20	0.057	0.025	18.091	0.581

Equation: EQ_AUTO_INTEGRA Workfile: ARIMA_F...

Correlogram of Residuals Squared

Date: 09/26/20 Time: 00:24
Sample: 2005M08 2020M07
Included observations: 179

	AC	PAC	Q-Stat	Prob
1	0.007	0.007	0.0092	0.924
2	0.032	0.032	0.1964	0.906
3	0.136	0.136	3.6118	0.307
4	-0.024	-0.026	3.7152	0.446
5	-0.015	-0.024	3.7575	0.585
6	0.116	0.101	6.2614	0.395
7	-0.023	-0.016	6.3568	0.499
8	-0.010	-0.013	6.3740	0.605
9	0.075	0.049	7.4368	0.592
10	-0.013	-0.004	7.4712	0.680
11	0.054	0.057	8.0305	0.711
12	0.004	-0.025	8.0339	0.782
13	-0.019	-0.014	8.1019	0.837
14	0.048	0.041	8.5479	0.859
15	0.038	0.031	8.8341	0.886
16	-0.005	0.000	8.8392	0.920
17	0.050	0.027	9.3374	0.929
18	0.131	0.129	12.773	0.805
19	0.025	0.032	12.902	0.844
20	0.068	0.036	13.835	0.839

Equation: EQ_AUTO_PRIMA Workfile: ARIMA_FO...

Correlogram of Residuals Squared

Date: 09/26/20 Time: 00:27
Sample: 2005M08 2020M07
Included observations: 178

	AC	PAC	Q-Stat	Prob
1	0.005	0.005	0.0054	0.942
2	0.046	0.046	0.3959	0.820
3	0.140	0.139	3.9615	0.266
4	-0.009	-0.012	3.9757	0.409
5	-0.015	-0.029	4.0183	0.547
6	0.092	0.075	5.5805	0.472
7	-0.027	-0.023	5.7184	0.573
8	-0.012	-0.014	5.7451	0.676
9	0.051	0.031	6.2308	0.717
10	-0.011	-0.002	6.2535	0.794
11	0.048	0.052	6.6936	0.823
12	0.001	-0.018	6.6939	0.877
13	-0.016	-0.015	6.7421	0.915
14	0.052	0.045	7.2766	0.924
15	0.018	0.015	7.3400	0.948
16	0.014	0.018	7.3775	0.965
17	0.035	0.014	7.6262	0.974
18	0.122	0.123	10.597	0.911
19	0.018	0.019	10.664	0.935
20	0.055	0.026	11.279	0.939

Figure 10. Validation of the squared residuals.

Source: Prepared by the author using Eviews 10.

deviates by 0.2796 units and in percentage terms or MAPE, the deviation is 1.20%. AFP Integra has the highest RMSE with respect to the other three AFPs, with a deviation of 2.8076 units. In percentage terms, AFP Prima has the highest deviation with 1.66%.

The forecasts with ARIMA were compared with the double exponential smoothing techniques contained in Table 3, which were automatically selected out of eight techniques by the Risk Simulator software for having the lowest error statistics; these techniques are contained in Table 3 and it is observed that ARIMA has lower forecast errors than double exponential smoothing.

Hypothesis Testing

Specific hypothesis 1: The trend of the monthly average of the quota values for each AFP of the type 2 fund directly influences the unit root.

Table 2. *Forecast Errors with ARIMA.*

	AFP	RMSE	MAE	MAPE	U-THEIL
In sample	Habitat	0.2796	0.1665	1.20%	0.8912
	Profuturo	2.6354	1.7516	1.61%	0.9027
	Integra	2.8076	1.8535	1.60%	0.9013
	Prima	0.5542	0.3582	1.66%	0.9222

Source: Prepared by the author.

Table 3. *Forecast Errors with Double Exponential Smoothing Techniques.*

	AFP	RMSE	MAE	MAPE	U-THEIL
In sample	Habitat	0.3046	0.0928	1.33%	0.9684
	Profuturo	2.7707	7.6767	1.72%	0.9767
	Integra	2.9413	8.6515	1.72%	0.9767
	Prima	0.5772	0.3332	1.78%	0.9782

Source: Prepared by the author.

Table 4. *Unit Root Hypothesis Tests for AFPs Habitat, Profuturo, Integra and Prima.*

Hypothesis test for AFP Habitat	Hypothesis test for AFP Profuturo
a) Null and alternate hypothesis H_0: $\phi = 1$; x_t has UR H_1: $\phi < 1$; x_t has no UR	a) Null and alternate hypothesis H_o: $\phi = 1$; x_t has UR H_1: $\phi < 1$; x_t has no UR
b) Significance level α = 0.05	b) Significance level α = 0.05
c) *p*-value = 0.9980	c) *p*-value = 0.0495
Decision: As *p*-value = 0.9980 > 0.05, H_0 is not rejected. The series has at least one UR.	Decision: As *p*-value = 0.0495 < 0.05, H_0 is rejected. The original has no UR.
Hypothesis test for AFP Integra	**Hypothesis test for AFP Prima**
a) Null and alternate hypothesis H_0: $\phi = 1$; x_t has UR H_1: $\phi < 1$; x_t has no UR	a) Null and alternate hypothesis H_0: $\phi = 1$; x_t has UR H_1: $\phi < 1$; x_t has no UR
b) Significance level α = 0.05	b) Significance level α = 0.05
c) *p*-value = 0.9944	c) *p*-value = 0.9927
Decision: As *p*-value = 0.9944 > 0.05, H_0 is not rejected. The series has at least one UR.	Decision: As *p*-value = 0.9927 > 0.05, H_0 is not rejected. The series has at least one UR.

Source: Prepared by the author.

Table 4 shows that the original series of AFP Profuturo does not show a trend and its *p*-value is equal to 4.95%, less than 5%, so the H_0 (which states that the series has a UR) is rejected and, therefore, the series is stationary. Table 4 also shows that the *p*-values of Habitat, Integra and Prima are above 5%, so the H_0s are not rejected since they have at least one UR and are not stationary.

Specific hypothesis 2: Stationarity has a direct influence on the mean of the return of the monthly average of the quota values for each AFP of type 2 fund.

A differentiation was made for each AFP, and the results of the hypothesis tests contained in Table 5 indicate that their *p*-values are less than 5% and, therefore, the null hypotheses are rejected. The differentiated series for each AFP has no unit root and is therefore stationary with constant mean.

Specific hypothesis 3: Stationarity directly influences the variance of the return of the monthly average of the quota values for each AFP of the type 2 fund.

It is verified that the differentiated series have no memory using the Ljung Box (LB) statistic for small samples. The joint tests for the squared residuals of the four AFPs present white noise, since all their *p*-values are above 5%, as shown in Table 6.

Specific hypothesis 4: There is a correlation between the observed values of the monthly average of the quota values for each AFP of the type 2 fund and the predicted values.

Table 7 shows that the results of the correlation coefficients of the observed values of the monthly average of the quota values for each AFP of the type 2 fund and the predicted values correspond to strong positive correlations; the table also contains their *p*-values.

In the hypothesis tests contained in Table 8, it is observed that the *p*-values of each AFP are below 5%, therefore, in addition to the strong and direct correlation between the observed and predicted values, these can be corroborated with the results of the error statistics contained in Table 3.

DISCUSSION

It was observed that the original series of the monthly average of the quota values for each type 2 fund

Table 5. *Hypothesis Test for the Mean of AFPs Habitat, Integra, Prima and Profuturo.*

Hypothesis test for AFP Habitat	**Hypothesis test for AFP Integra**
a) Null and alternate hypothesis H_0: $\phi = 1$; x_t has UR H_1: $\phi < 1$; x_t has no UR	a) Null and alternate hypothesis H_0: $\phi = 1$; x_t has UR H_1: $\phi < 1$; x_t has no UR
b) Significance level α = 0.05	b) Significance level α = 0.05
c) *p*-value = 0.000	c) *p*-value = 0.000
Decision: As *p*-value = 0.000 < 0.05, H_0 is rejected. The differentiated series has no RU and is therefore stationary.	Decision: As *p*-value = 0.000 < 0.05, H_0 is rejected. The differentiated series has no RU and is therefore stationary.
Hypothesis test for AFP Prima	**Hypothesis test for AFP Profuturo**
a) Null and alternate hypothesis H_0: $\phi = 1$; x_t has UR H_1: $\phi < 1$; x_t has no UR	a) Null and alternate hypothesis H_0: $\phi = 1$; x_t has UR H_1: $\phi < 1$; x_t has no UR
b) Significance level α = 0.05	b) Significance level α= 0.05
c) *p*-value = 0.000	c) *p*-value = 0.000
Decision: As *p*-value = 0.000 < 0.05, H_0 is rejected. The differentiated series has no RU and is therefore stationary.	Decision: As *p*-value = 0.000 < 0.05, H_0 is rejected. The differentiated series has no RU and is therefore stationary.

Source: Prepared by the author.

Table 6. *Hypothesis Test for the Variance of AFPs Habitat, Integra, Prima and Profuturo.*

Hypothesis test for AFP Habitat	**Hypothesis test for AFP Integra**
a) Null and alternate hypothesis H_0: $\phi 1 = \phi 2 = \dots = \phi n$; x_t has no white noise H_1: at least one is different	a) Null and alternate hypothesis H_0: $\phi 1 = \phi 2 = \dots = \phi n$; x_t has no white noise H_1 at least one is different
b) Significance level α = 0.05	b) Significance level α = 0.05
c) *p*-values > 0.05	c) *p*-values > 0.05
Decision: As *p*-values > 0.05, H_0 is not rejected. The differentiated series has white noise and, therefore, homoscedastic variance.	Decision: As *p*-values > 0.05, H_0 is not rejected. The differentiated series has white noise and, therefore, homoscedastic variance.
Hypothesis test for AFP Prima	**Hypothesis test for AFP Profuturo**
a) Null and alternate hypothesis H_0: $\phi 1 = \phi 2 = \dots = \phi n$; x_t has no white noise H_1: at least one is different	a) Null and alternate hypothesis H_0: $\phi 1 = \phi 2 = \dots = \phi n$; x_t has no white noise H_1: at least one is different
b) Significance level α = 0.05	b) Significance level α = 0.05
c) *p*-values > 0.05	c) *p*-values > 0.05
Decision: As *p*-values > 0.05, H_0 is not rejected. The differentiated series has white noise and, therefore, homoscedastic variance.	Decision: As *p*-values > 0.05, H_0 is not rejected. The differentiated series has white noise and, therefore, homoscedastic variance.

Source: Prepared by the author.

AFP had a trend and, therefore, were not stationary, so logarithmic differentiations were made to convert them. This is consistent with the results of the following research:

- Villalba and Flores (2016) studied the price and quotes index (IPC) of the Mexican stock market and verified the behavior of the volatility and the importance of the stationarity of its series for forecasting the prices of the stocks that compose it.
- Parody et al. (2016) calculated the daily closing prices of the shares of Banco de Colombia, Banco de Bogotá and Banco de Occidente between July 17 and 24, 2015, and obtained *los rendimientos diarios de las series de cada banco estudiado mediante la diferencia obtenida*

Table 7. *Results of Pearson's correlation coefficient.*

Correlation Probability	HABITAT_obs	HABITAT_pron
HABITAT_obs	1.0000	
HABITAT_pron	0.9923	1.0000
P value	0.0000	-----

Correlation Probability	INTEGRA_obs	INTEGRA_pron
INTEGRA_obs	1.0000	
INTEGRA_pron	0.9971	1.0000
P value	0.0000	-----

Correlation Probability	HABITAT_obs	HABITAT_pron
PRIMA_obs	1.0000	
PRIMA_pron	0.9968	1.0000
P value	0.0000	-----

Correlation Probability	PROFUTU_obs	PROFUTU_pron
PROFUTU_obs	1.0000	
PROFUTU_pron	0.9971	1.0000
P value	0.0000	-----

Source: Prepared by the author.

Note: the results are presented as reported by the software.

Tabla 8. *Hypothesis tests for Pearson’s correlation of AFP Habitat, Integra, Prima and Profuturo.*

Hypothesis test for AFP Habitat	**Hypothesis test for AFP Integra**
a) Null and alternate hypothesis H_0: $\rho = 0$ There is no correlation between the observed and predicted values. H_1: $\rho \neq 0$ There is a correlation between the observed and predicted values.	a) Null and alternate hypothesis H_0: $\rho = 0$ There is no correlation between the observed and predicted values. H_1: $\rho \neq 0$ There is a correlation between the observed and predicted values.
b) Significance level α = 0.05	b) Significance level α = 0.05
c) *p*-value = 0.0000	c) *p*-value = 0.0000
Decision: As *p*-value 0.0000 < 0.05, H_0 is rejected. There is a direct correlation between the observed and forecast values.	Decision: As *p*-value 0.0000 < 0.05, H_0 is rejected. There is a direct correlation between the observed and forecast values.
Hypothesis test for AFP Prima	**Hypothesis test for AFP Profuturo**
a) Null and alternate hypothesis H_0: $\rho = 0$ There is no correlation between the observed and predicted values H_1: $\rho \neq 0$ There is a correlation between the observed and predicted values.	a) Null and alternate hypothesis H_0: $\rho = 0$ There is no correlation between the observed and predicted values H_1: $\rho \neq 0$ There is a correlation between the observed and predicted values.
b) Significance level α = 0.05	b) Significance level α = 0.05
c) *p*-value = 0.0000	c) *p*-value = 0.0000
Decision: As *p*-value 0.0000 < 0.05, H_0 is rejected. There is a direct correlation between the observed and forecast values.	Decision: As *p*-value 0.0000 < 0.05, H_0 is rejected. There is a direct correlation between the observed and forecast values.

Source: Prepared by the author.

entre los logaritmos neperianos de los precios actuales y los precios del día inmediatamente anterior [the daily returns of the series of each bank studied through the difference of the neperian logarithms of the current prices and the prices of the immediately preceding day].

It was also observed that the stationary series of the monthly average of the quota values for each AFP of the type 2 fund presents constant or homogeneous variance, which means that it does not present much volatility, according to the following findings:

- Amaris et al. (2017) conclude that *el análisis estadístico permitió tomar una decisión del modelo escogido, el cual cumple con los parámetros requeridos de normalidad, varianza constante y aleatoriedad* [The statistical analysis made it possible to make a decision on the chosen model, which complies with the required parameters of normality, constant variance and randomness].
- Gallego-Nicasio et al. (2018) found in one of their results that, when performing the first differentiation, the new series is stationary, homogeneous and integrated of order one. They say that the ARIMA (p,d,q) model is called Autoregressive Integrated Moving Average process of order p, d, q; and that the disturbance or error is known as white noise, with the mean being zero, the variance homocedastic and the covariance null among the shocks or errors of the observations.

CONCLUSIONS

1. The original series of the monthly average of the AFP quota values of the type 2 fund, which began in December 2005, shows an upward trend during the period 2005-2020.

2. In order to forecast the monthly average of the AFP quota values of the type 2 fund with the Box and Jenkins or ARIMA models, the trends must be eliminated by differentiation until the series becomes stationary. In this case, only the first differentiation was enough.

3. The results show that the series corresponds to a stochastic process in the weak sense because both the first and second moments of the series are invariant over time.

4. The returns were calculated with the logarithmic differentiation of the current month average and the previous month average to make them both stationary.

5. The return models depend of a mean, which is its long-term behavior, plus an error or disturbance that deviates this behavior; however, these errors are normally distributed and, therefore, the variance is homoscedastic.

6. With the correlograms, the Ljung Box statistic and the *p*-value, it was validated that the original series of the monthly average of the quota values for each AFP of the type 2 fund had memory and it was concluded that it is homoscedastic, or of constant variance, over time, so it can be forecast with the Box and Jenkins methodology.

7. The residuals and squared residuals have white noise and their variance is homoscedastic, so the Box and Jenkins methodology can be used.

8. Since they have constant variance and are not highly volatile, the returns are conservative and therefore do not meet the expectations of workers.

9. Economic and financial crises negatively impact the investment returns of workers.

10. The forecasts of the samples using the Box and Jenkins methodology have lower forecast errors than when using double exponential smoothing.

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